Worked Solutions

Edexcel C3 Paper A

1. $\text{fg}(x) = \text{g}^{-1}\text{g}(x) = x.$ **(3)**

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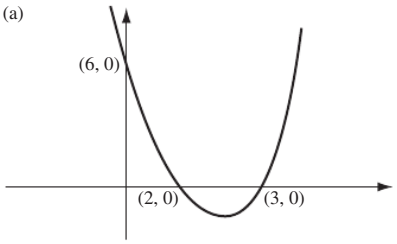
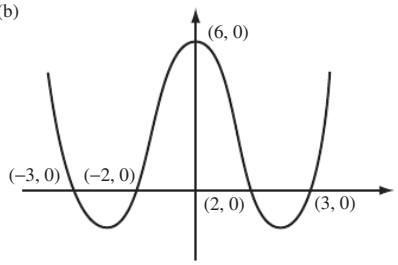
2. (a)



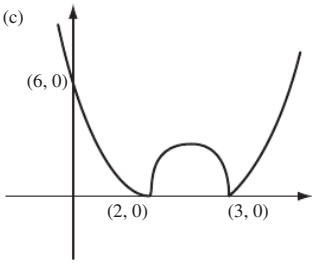
(2)

(b)



(3)

(c)



(3)

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3. $\text{e}^{2x} - 7\text{e}^{x} + 12 = 0$

$(\text{e}^{x} - 3)(\text{e}^{x} - 4) = 0$

$\text{e}^{x} = 3 \Rightarrow x = \ln 3$

$\text{e}^{x} = 4 \Rightarrow x = \ln 4$

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4. $\sin A = \frac{3}{5} \Rightarrow \cos A = -\frac{4}{5}$

$\sin B = \frac{5}{13} \Rightarrow \cos B = -\frac{12}{13}$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$= \frac{3}{5} \cdot \frac{-12}{13} - \frac{-4}{5} \cdot \frac{5}{13} = -\frac{16}{65}$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{-\frac{3}{4} - \frac{5}{12}}{1 - \left(-\frac{3}{4}\right)\left(-\frac{5}{12}\right)}$$

$$= \frac{-\frac{14}{12}}{\frac{11}{16}} = \frac{-56}{33}$$

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5. $\frac{\text{d}y}{\text{d}x} = x\text{e}^{x} + \text{e}^{x}$

at turning point, $(x + 1)\text{e}^{x} = 0 \Rightarrow x = -1$

$y = -\text{e}^{-1}$ pt. $\left(-1, \frac{-1}{\text{e}}\right)$

$\frac{\text{d}^2y}{\text{d}x^2} = \text{e}^{x} + x\text{e}^{x} + \text{e}^{x}$

$x = -1$ $\frac{\text{d}^2y}{\text{d}x^2} = \text{e}^{-1} > 0$ $\therefore$ minimum

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6. (*a*)

$$\left.\begin{aligned} f(1) &= 1-6+7=2 \\ f(2) &= 4-12+7=-1 \end{aligned}\right\} \text{change of sign}$$

(2)

(*b*) $7 = 6x - x^2$

$7 = x(6-x)$

$x = \dfrac{7}{6-x}$ **(2)**

(*c*) $x_1 = 1.75,\ x_2 = 1.6470,\ x_3 = 1.6081,\ x_4 = 1.5938,\ x_5 = 1.5886,\ x_6 = 1.5868$

Ans. $x = 1.59$ (3 s.f.) **(3)**

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7. $\dfrac{dx}{dy} = -3\sin 3y,$

$\dfrac{dy}{dx} = -\dfrac{1}{3\sin 3y}$

$y = \dfrac{\pi}{6},\ \dfrac{dy}{dx} = -\dfrac{1}{3}$

$y = \dfrac{\pi}{6},\ x = 0.$

$y - \dfrac{\pi}{6} = -\dfrac{1}{3}(x-0)$

$y - \dfrac{\pi}{6} = -\dfrac{1}{3}x$

$6y + 2x - \pi = 0$ **(8)**

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8. (*a*) $\dfrac{3}{(x+2)(x+3)} - \dfrac{2}{x+3} = \dfrac{1}{x+2}$

$\dfrac{3-2(x+2)}{(x+2)(x+3)} = \dfrac{x+3}{(x+2)(x+3)}$

$3 - 2x - 4 = x + 3$

$-4 = 3x$

$x = -\dfrac{4}{3}$

(*b*) $\dfrac{(2x-3)(2x+3)}{(x+1)\left(x^2-x+1\right)} \times \dfrac{x+1}{(2x+3)(x-5)}$

$\dfrac{2x-3}{\left(x^2-x+1\right)(x-5)}$

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9. (*a*) $f(1) = 1 - 1 - 3 + 3 = 0.$

$(x-1)\left(x^2-3\right) = 0 \qquad x = \pm\sqrt{3}$

(*b*) $\tan^3\theta - 3\tan\theta + 4 = 1 + \tan^2\theta$

$\tan^3\theta - \tan^2\theta - 3\tan\theta + 3 = 0$

same as (i) with $x = \tan\theta$

(*c*) $\tan\theta = 1 \Rightarrow \theta = \dfrac{\pi}{4}$

$\tan\theta = \sqrt{3} \Rightarrow \theta = \dfrac{\pi}{3}$

$\tan\theta = -\sqrt{3} \Rightarrow \theta = \dfrac{2\pi}{3}$

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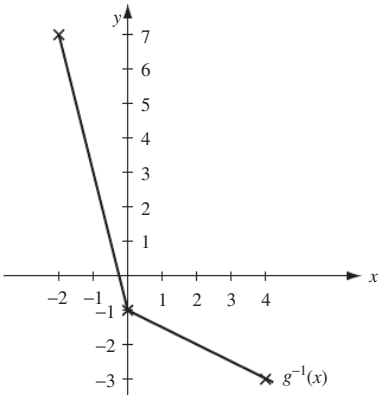
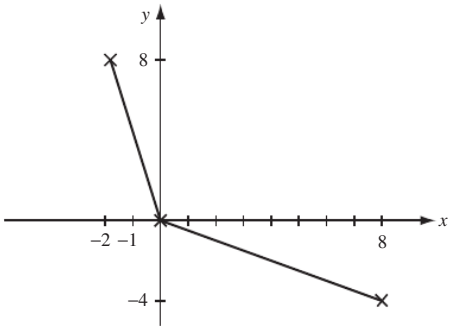
10. (*a*)



(3)

(*b*)



(3)

(*c*) $g(-3) = 4$

$hg(-3) = h(4)$

$= -2$ **(3)**
